# Physics 143b: Honors Waves, Optics, and Thermo

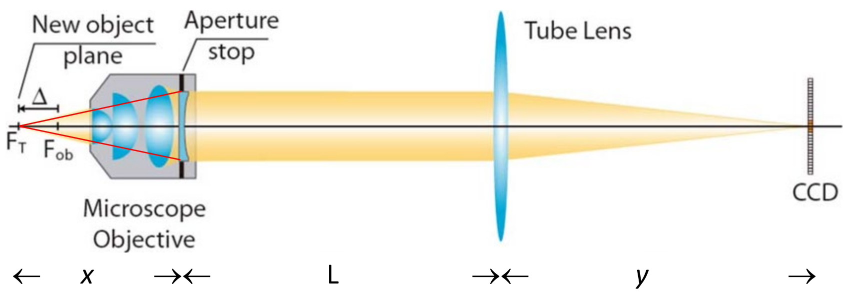
Spring Quarter 2021

Problem Set #7

Due: 11:59 pm, Saturday, May 22. Please submit to Canvas.

1. Optical microscope. (5 points each)

   A generic design of an optical microscope is illustrated below




We consider an object on the left side of the microscope objective with the working distance of *x*, the distance between the object and the objective.) For simplicity, we assume the objective is a thin lens, and it collimates the light coming from the object onto the tube lens. The tube lens then forms an image on the CCD. Assume the objective has a focal length of $f_1$ and the tube lens has a focal length of $f_2$ .

a) Use the lens equation $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}$ and show that when $x = f_1$, the CCD should be placed away from the tube lens by exactly $y = f_2$ to form a clear image. The distance between the two lenses *L* does not matter. (This is called infinite conjugation). Draw the ray diagram for light coming from the object and show that rays do converge to the CCD.

   (Hint: For right-propagating rays, use the convention that $D_0$ is defined to be positive if the object is on the left of the lens and $D_i > 0$ if the image on the right of the lens.)

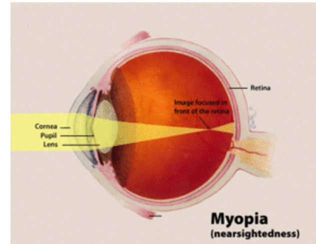
b) Under the condition $x = f_1$ and $y = f_2$ Draw the ray diagram for an object slightly off the image axis and show that the image on the CCD is upside down and is magnified by a factor of $M_0 = f_2/f_1$.

   (Hint: Magnification of a single lens is given by $M = -D_i/D_0$. For compound lenses, it is the product of all magnifications.)

c) When the working distance deviates from the above condition slightly $x = f_1 + dx$, where $dx \ll f_1, f_2, L$. (This frequently happens in real experiments), one would need to move the position of CCD to $y = f_2 + dy$ in order to get a clear image again? Determine *dy* in terms *dx.*

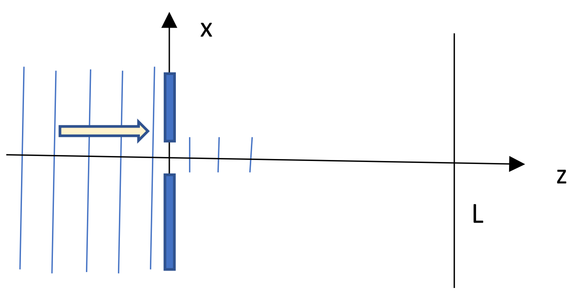
d) Following c), when the image is reoptimized, how much is the magnification modified $M = M_0 + dM$? Determine *dM* in terms of *dx* and show such correction to the magnification vanishes when $L = f_1 + f_2$.

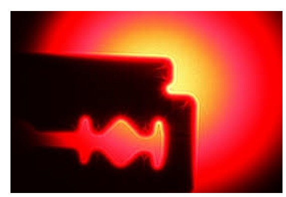
2. Myopia correction (8 points)
Myopia occurs when the incident light from a point source is focused by the lens in front of the retina, see figure, while a healthy eye focuses it exactly on the retina.
The distance between the lens and the retina is effectively 25mm (it is effective so we may consider the index of refraction $n \approx n_{air} \approx 1$ behind the lens of the eyeball.).




a) What would be the focal length of the lens of a healthy person when he/she is reading a book 25 cm away from the eyes.
b) John's reading eyeglasses has a refractive power of -5.75 diopters (D), which is defined as the reciprocal of the focal length in meter, namely, $f = \frac{1}{-5.75} m$, and it is placed 10mm in front of his eyeballs. Show that it effectively brings the book 11.08 cm in front of his eyes.
c) Show that the focal length of his eyeball lens is 2.33 mm shorter than that of a healthy person.

3. Fraunhofer diffraction (8 points each)
Here we will treat optical diffraction more rigorously based on the wave equation we learned. Let's assume we have a uniform plane electromagnetic wave propagating in the z direction with electric field $E(x,z,t) = E_0 e^{i(kz-\omega t)}$, where $k = \omega/c$ is the wave number, $\omega$ is the angular frequency and $c$ is the speed of light. The light is blocked by an aperture at z=0 such that the electric field immediate after the aperture is given by $E(x,0^+,t) = E_{in}(x)e^{i(kz-\omega t)}$. The detector (screen) is located at $z = L$.



The general solution of the electric field can be determined from the solution of the wave equation as the sum of spherical waves originated from all locations at $(-\infty < x' < \infty, z = 0^+)$ with non-zero electric field $E_{in}(x')$, namely,

$$E(x, z > 0) = \int E_{in}(x') \frac{e^{i\,(kr\prime - \omega t)}}{r'}\, dx',$$

where $r' = |\vec{r}'|$ and $\vec{r}' = (x - x', z)$ is the vector that indicates propagation from a point source at $(x', 0)$ to a detector at $(x, z)$. In the integrand, $\frac{e^{i(kr'-\omega t)}}{r\prime}$ can be considered as the Green function of the point source.

a) Assume the screen is far from the aperture $L \gg x$ and $L \gg x'$, show that the electric field on the screen can be approximated as

$$E(x, L) \approx \frac{1}{L} e^{i(kL-\omega t)} \int E_{in}(x') e^{\frac{ik(x-x')^2}{2L}} dx'.$$

Let's analyze the ideal 2-slit interference pattern by considering light passing two infinitesimal slits separated by $d$ with electric field $E_{in}(x) = E_0 \delta\left(x - \frac{d}{2}\right) + E_0 \delta(x + \frac{d}{2})$. Determine the electric field on the screen. Identify the maxima and minima of the intensity on the screen. Show that the predicted locations of constructive interferences are consistent with our derivation in the class/in the textbook: $d \sin\theta = n\lambda$, where $\sin\theta \approx x/L$?

b) Now we consider single-slit diffraction. Assume that the incident electric field $E_{in}\left(-\frac{d}{2} < x < \frac{d}{2}\right) = E_0$ is a constant within the width of the slit $d$ and is zero elsewhere. Determine the electric field on the screen $E(x, L)$. Are the maximum and minimum positions consistent with the what we derived in the class / textbook?

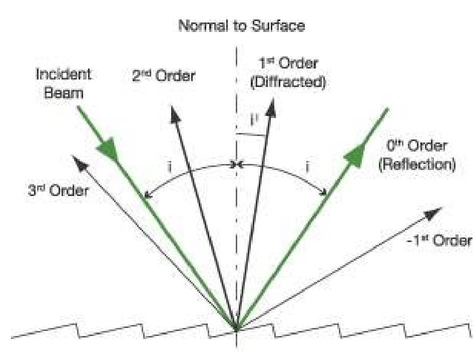
(Hint: you may assume $d \ll x$ and thus approximate $(x - x')^2 \approx x^2 - 2x'x$ in order to compare with the lecture note/textbook.)

c) Next we consider edge diffraction, where half the space is blocked by the aperture $E_{in}(x) = E_0 H(x)$, where $H(x > 0) = 1$ and $H(x < 0) = 0$ is the Heaviside step function. Determine the electric field on the screen $E(x, L)$.

(Hint: Analytic expression requires Fresnel integrals. You may also sketch the intensity pattern.)

d) Use the result of c) and comment on why when you cannot see someone around the corner, you could still hear his/her voice? Don't sound and light all propagate as waves and thus both should reach you?

4. Diffraction grating (8 points each)
Diffraction grating is the critical component behind a large number of optical devices, including DVD, spectrometers, digital displays and so on. Grating is a reflective surface with periodic structure, see figure. Assuming the periodicity of the structure is *d* and a beam with wavelength $\lambda$ illuminates the grating with an incident angle $i$. Show that in addition to the regular reflection, we have diffracted beams with different orders propagating in different directions.



a) The *n*-th order refracted beam originates from the constructive interferences of scattered beams with reflection angle $\theta_n$, where $n = 0, \pm 1, \pm 2$ ... In particular, the regular reflected beam can be considered as the 0th-order refraction with $\theta_0 = i$. Show that

$$d(\sin i - \sin \theta_n) = n\lambda,$$

b) German scientist Joseph von Fraunhofer first noted the sodium absorption in the solar spectrum. The Sodium absorption contains two very closely spaced lines near $\lambda \approx$589.0 and 589.6nm, which can be distinguished from their slightly different diffraction angle. Consider the sunlight goes straight down to the grating (incident angle $i = 0$) with periodicity *d*=(1/1200) mm, calculate the angles of the 1st order diffraction $\theta_1$ and show that for small wavelength difference $\Delta\lambda$, we have

$$\frac{\Delta\theta}{\theta} = -\frac{\Delta\lambda}{d}\left(1 - \frac{\lambda^2}{d^2}\right)^{-1/2}$$

c) Diffraction gratings are also used in quantum optics experiments to fine tune the laser wavelength at the precision of 10 fm ($10^{-14}$m) by optical feedback. Let's consider a laser beam is oriented toward the grating and feedback occurs when the first order diffraction goes exactly back toward the laser source $\theta_1 = -\theta_i$. How would you set the incident angle $\theta_i$ of a laser beam toward a grating with 1800 lines per meter to stabilize a laser at the wavelength $\lambda$ =852 nm. Show that you can fine tune the laser wavelength $d\lambda$ by rotating the diffraction grating angle by $d\phi$. Determine the tuning sensitivity $d\lambda/d\phi$ of the laser.