Examples of Fourier spectra

- Trumpet, piano, violin, flute
- Seismic waves
- Telecommunication spectrum
- Cosmic microwave background
- Optical fiber noise
- Spontaneous formation of patterns in quantum gas
Connecting two strings of diff. density \( \rho_1, \rho_2 \)

\[ v_1 = \frac{\omega_1}{k_1}, \quad v_2 = \frac{\omega_2}{k_2} \]

Waves going to diff. medium

\( \text{water, light, seismic and others.} \)

What do we do at the boundary?

**General consideration:** \( \omega_1 = \omega_2 \) and \( T_1 = T_2 \) \( \Rightarrow \) \( k_1 : k_2 = T_1 : T_2 = v_2 : v_1 \)

\[ \varphi(x) = \begin{cases} A e^{i(k_1 x - \omega t)} + B e^{-i(k_1 x - \omega t)} & \text{incident} \\ C e^{i(k_2 x - \omega t)} & \text{transmission} \end{cases} \]

Assume \( x_c = 0 \).

\[ \varphi(x < 0) = \varphi(x > 0) \]

Continuity:

\[ \varphi_L(x_c, t) = \varphi_R(x_c, t) \]

No force at the interface:

\[ \varphi_L'(x_c, t) = \varphi_R'(x_c, t) \]

\[ \varphi(0^-) = \varphi(0^+) \Rightarrow A + B = C \]

\[ \varphi(0^-) = \varphi(0^+) \Rightarrow ik_1 (A - B) = i k_2 C \]

\[ \Rightarrow \begin{cases} A + B = C \\ A - B = \frac{k_2}{k_1} C \end{cases} \]

Reflection:

\[ \frac{B}{A} = \frac{\sqrt{k_1} - \sqrt{k_2}}{\sqrt{k_1} + \sqrt{k_2}} = \frac{\frac{\sqrt{\rho_1} - \sqrt{\rho_2}}{\sqrt{\rho_1} + \sqrt{\rho_2}}}{\sqrt{\rho_1} + \sqrt{\rho_2}} = \frac{v_2 - v_1}{v_2 + v_1} \]

Transmission:

\[ \frac{C}{A} = \frac{-2k_1}{k_1 + k_2} = \frac{\frac{2\sqrt{\rho_1}}{\sqrt{\rho_1} + \sqrt{\rho_2}}}{\sqrt{\rho_1} + \sqrt{\rho_2}} = \frac{2v_2}{v_2 + v_1} \]

Check simple limits:

1. \( \rho_1 = \rho_2 \) (Reflection \( B \to 0 \) \( C \to 1 \) \( \checkmark \))
2. \( \rho_2 \gg \rho_1 \) (hitting a wall) \( C \to 0 \) \( B = -1 \) \( \checkmark \)
3. \( \rho_1 \gg \rho_2 \) (hitting nothing) \( C \to 0 \) \( B = 1 \) \( \checkmark \)