Physics 143A: Honors Waves, Optics, and Heat

Spring Quarter 2025

Final Exam Practice

Formula sheet

- Sound wave equation: $\partial_t^2\psi = \frac{1}{n\beta}\partial_x^2\psi = v^2\partial_x^2\psi$
- EM waves: $\partial_t^2 E = \frac{c^2}{n^2}\nabla^2 E, \partial_t^2 B = \frac{c^2}{n^2}\nabla^2 B$
- Doppler effect for wavs in a medium: $\omega_r = \omega_s\frac{v-v_s}{v-v_r}$
- Doppler effect for light in vacuum: $\omega_r = \omega_s\sqrt{\frac{v+\Delta v}{v-\Delta v}}, \Delta v = v_r - v_s$
- Snell's law: $n_1 sin\theta_1 = n_2 sin\theta_2$
- Thin lens formula: $\frac{1}{p}+\frac{1}{q} = \frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{1}{f}$
- Ideal gas law: $PV = Nk_BT$
- Ideal gas energy: $U = \gamma Nk_BT$
- 1st law of thermodynamics: $dU = TdS - PdV, S = k_B \ln\Omega$
- 2nd law of thermodynamics: $S(B) - S(A) \geq \int_A^B \frac{dQ}{T}$.
- 3rd law of thermodynamics: $\lim_{T\to 0} S(T) = 0$.

Problem 1 Please circle your choices

A. $\vec{A} = (A_x, A_y, A_z)$ is a vector field. Show that $\vec{\nabla}(\vec{A} \cdot \vec{A})$ equals to

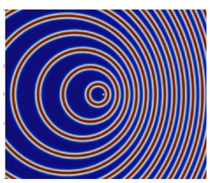
1. $2(\vec{\nabla}A_x, \vec{\nabla}A_y, \vec{\nabla}A_z) \cdot \vec{A}$ 2. $2(A_x\partial_x A_x + A_y\partial_y A_y + A_z\partial_z A_z)$ 3. $(2\,\vec{A} \cdot \vec{\nabla})\vec{A}$

B. Given the sound wave wavefunction $\psi = A\sin[k(x - vt)]$, which of the following is correct?

1. ψ indicates how far air molecules move perpendicular to the wave propagation direction.
2. The wave is propagating in the - x direction with frequency v.
3. The wave leads to a density wave with highest density at $x - vt = \frac{\lambda}{2}, \frac{3\lambda}{2} \dots$
4. The period of the sound wave is given by $\frac{2\pi}{kv}$.

C. The dispersion of a medium can be Taylor expanded as $\omega(k) = \omega'(0)k + \frac{1}{2}\omega''(0)k^2$, where $\omega'(0)$ and $\omega''(0)$ are both positive. Consider a wave propagating in the medium with wavenumber k_0.

1. The phase and group velocities are $v_p = \omega'(0)$ and $v_g = \omega'(0) + \omega''(0)k_0$.
2. The group velocity is greater than the phase velocity.
3. Waves at higher frequencies propagate faster.
4. In a medium particles can move faster than speed of light.

D. 

The diagram shows the wave fronts of a sound source.

1. The source is obviously moving to the left or wind is blowing to the right.
2. A static receiver on the right would hear a higher pitch than on the left.
3. If the receiver on the right approaches the source at the speed of the sound, the frequency he/she hears will appear to be 0.
4. This picture does not hold for electromagnetic waves since speed of light c is constant.

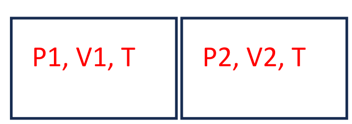
E. You find a magnifier made of a convex lens with focal length *f* =10 cm.

1. If you wish to start a fire by focusing the sunlight, you put the magnifier 2*f* from the leaves.
2. To read small fonts on the book, you put the magnifier at least 1*f* from the book.
3. You wish to use it under, the apparent focal length would be shorter but remains positive.
4. Magnifier can either magnify or demagnify an object when it forms an image on the screen.

F. In the double-slit experiment, the slits are separated by *d=0.01 mm* and the wavelength of the light is $\lambda = 10^{-3}mm$. We see interference fringes on a screen 1m away from the slit. Select true statements.

1. The number of fringes increases with larger wavelength.
2. There are only finite number of bright lines (constructive interferences) on the screen.
3. If we decrease the slit separation *d*, there will be denser fringes on the screen.
4. If we block one of the slits, there will be no more fringes on the screen.

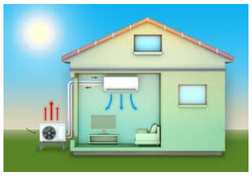
G. Two boxes of ideal gas with pressure $p_1 > p_2$ and volume $V_1 \neq V_2$ are isolated from the environment and separated by a thermally conducting divider that maintains the same temperature T, see figure. Now we allow system 1 to expand a bit with $dV_1 = -dV_2 > 0$ by slowly moving the divider. Choose the right statement:



1. Heat flows from system 1 to system 2 in order to maintain the balance of the temperature.
2. Entropy cannot stay constant and must increases during the process.

3. Both systems 1 and 2 become colder (lower temperature) after the process.

4. The process is irreversible. Even we move the divider back, the systems cannot recover their initial pressures and temperature.

H. 

The air conditioning maintains the lecture room KPTC at the temperature of $T_1=27^oC=300K$ while the temperature outside is $T_2=32^oC$. 30 students are in the room with average body temperature $T_3=36\,^oC$. Each student generates about 100 W=100 Joule/s of heat. Ignore other heat loss. Choose the correct statement:

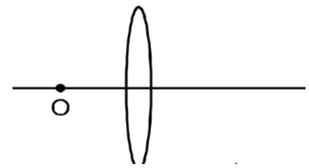
1. The AC absorbs 3000W from people and thus deposits 3000W to the environment.
2. An ideal AC would require (T_2/T_1-1) 3000W = 50W to remove 3000W of heat.
3. An ideal AC would require at least electric power of 3000W to remove 3000W of heat.
4. Even for an ideal AC, the total entropy increase of the system is at least (1/T3-1/T2)3000W.

I. In which thermodynamic process is no heat exchanged with the surroundings?

1. Isothermal
2. Adiabatic
3. Isobaric
4. Isochoric

Problem 2 Geometrical Optics

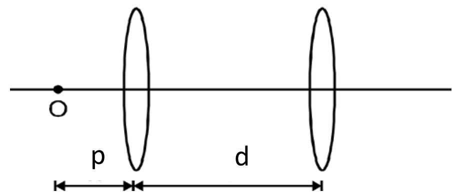
Compound lenses are widely used in optical instruments. Here we will study the general properties of single-lens and two-lens system.



Single lens: Consider a point light source is placed at O and the lens with focal length f is placed at p away from source. We can use the lens equation $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ to determine the position of the image q. Show that

a) For a single convex lens with $f>0$, show that we get an imaginary image with positive magnification $M=-\frac{q}{p}=\frac{f}{f-p}>0$ when $p<f$.
Draw light rays for the case of $p=f/2$ and show that you get an imaginary image at $q=-f$ with the magnification of $M=2$.

b) Now we move on to the two-lens system, where the second lens with focal length f' is placed d away from the first one, see figure. The image of the first lens acts as the object for the second lens. Derive the object distance p' and the image distance q' with respect to the second lens. Express your answers in p, d, f and f'.



c) In the case of $d \ll p$, the two lenses can be considered as one compound lens with an effective focal length f_{eff} such that $\frac{1}{p}+\frac{1}{q'}=\frac{1}{f_{eff}}$. Prove that

$$f_{eff}=\frac{ff'}{f+f'}\,.$$

Problem 3 Thermodynamical process and heat engine

Consider an ideal gas initially prepared in a container of volume V_0 with pressure P_0 and temperature T_0. Consider the following cycling processes and express all your results in terms of V_0, P_0 and T_0.

Step 1: the system is isothermally expanded by a factor of 2 to $2V_0$.
Step 2: the system is isochroically heated until it recovers its initial pressure P_0.
Step 3: the system is isobarically compressed from $2V_0$ to V_0.

a. Draw the 3 processes in the pressure-volume diagram. Use arrows to indicate the direction of the processes and the pressure and volume after each process.

b. Calculate the internal energy U, pressure P, volume V and temperature T in the beginning of each step and fill the following table:

Step	Internal energy U	Pressure P	Volume V	Temperature T
1	$\gamma P_0 V_0$	P_0	V_0	T_0
2				
3				

c. List the amount of work the system does to the environment and the heat the system absorbs from the environment during each step. (Hint: use 1st)

Step	Work done $\Delta W = \int P dV$	Heat absorbed ΔQ	Entropy increased ΔS
1			
2			
3			
Sum 1+2+3			

d. Given the total amount of work the system does and the heat system absorb or deposit to the environment, describe the overall function of the system. Is it more like a heat engine, a refrigerator or something else?