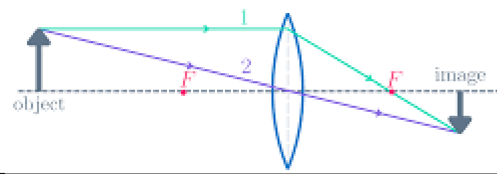
Physics 143b: Honors Waves, Optics, and Heat

Spring Quarter 2025
Problem Set #8
Due: 11:59 pm, Wednesday, May 20. Please submit to Canvas.

1. Geometrical Optics in daily life (10 points each)
   Let's do some practices to make sure we can calculate the position of images and magnification after an optical system.
   A. Curved interface In a sunny day you are gently tapping the water in a swimming pool. Suddenly you notice sunlight is focused on the bottom of the pool by the ripple. Assume the pool is 1.5 m in depth, and the ripple has a wavelength of λ=10 cm, how high would the amplitude of the ripple be such that the sunlight is well focused on the bottom?
   (Hint: you may assume the sun is right above the pool, the water index of refraction is n=1.3. Sun light focuses when it passes through the crest of the ripple. You may expand the depth of the water $D(x) = D_0 + A\cos kx$ surface depth of the water near the crest $x = 0$.)

   B. Magnification of a single lens You found a random magnifier in a garage sale. To find its focal length you use it to project your cell phone on the wall. You did the following tests:

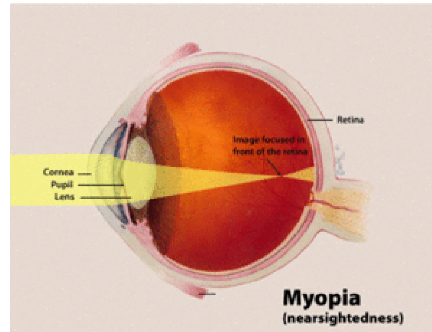


	Distance to object P (cm)	Distance to image Q (cm)	Magnification M
Exp 1	**10**	**Cannot find image**	
Exp 2	**30**	**42**	
Exp 3	**15**	**Cannot find image**	

   Determine the focal length, explain where images are in Exps 1 and 3, and compute the magnifications in the last column.
   (Hint: From the diagram, you can see the magnification is simply given by $M = -\frac{Q}{P}$.)

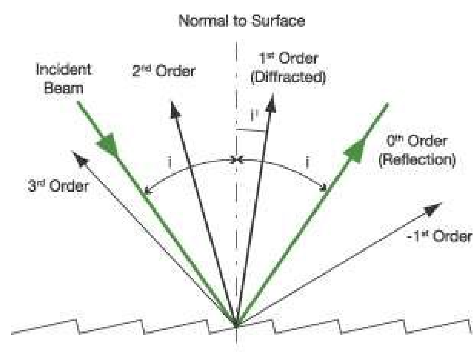
   C. Compound lenses You want to build a simple telescope. Here is the plan:
   Lens 1 (Objective) f1=+50 cm
   -- To collect light from a distant object and form a real, inverted image of the object
   Lens 2 (Field lens): f2=+5 cm, 45 cm from lens 1
   -- redirect and shape the light rays from the intermediate image
   Lens 3 (Eyepiece): f3=+10 cm, 10 cm from lens 2
   -- magnify the intermediate image formed by the objective
   Please (a) provide a ray diagram illustrating the path of light through the lens system
   (b) find the location of the final image with respect to the eyepiece, and
   (c) calculate the angular magnification of the system.
   (Hint: Overall magnification is the product of the magnifications of individual lenses.)

2. **Myopia correction** (10 points each)
Myopia occurs when the incident light from a point source is not properly focused on the retina, but in front of the retina, see figure, while a healthy eye focuses it on the retina.
The distance between the lens and the retina is effectively 25 mm (it is effective so we may consider the index of refraction $n \approx n_{air} \approx 1$ behind the lens of the eyeball.).



   a) What would be the focal length of the lens of a healthy person when he/she is reading a book 25 cm away from the eyes, called *distance of distinct vision*.
   b) Olivia's reading eyeglasses has a refractive power of -5.75 diopters (D), which is defined as the reciprocal of the focal length in meter, namely, $1/f = -5.75/m$, and it is placed 10mm in front of his eyeballs. Show that it effectively brings the book 14 cm closer to her eyes.
   c) Show that the focal length of her lens is 2.33 mm shorter than that of a healthy person.

3. **Diffraction grating** (10 points each)
Diffraction grating is the critical component behind a large number of optical devices, including DVD, spectrometers, digital displays and overhead projector, called digital micromirror device. Fundamentally grating is a reflective surface with periodic structure, see figure. Assuming the periodicity of the structure is *d* and a beam with wavelength λ illuminates the grating with an incident angle i. Show that in addition to the regular reflection, we have diffracted beams with different orders propagating in different directions.



   a) The *n*-th order refracted beam originates from the constructive interferences of scattered beams with reflection angle θ_n, where $n = 0, \pm1, \pm2 \ldots$ In particular, the regular reflected beam can be considered as the 0th-order refraction with $\theta_0 = i$. Show that

$$d(\sin i - \sin \theta_n) = n\lambda,$$

   b) German scientist Joseph von Fraunhofer first noted the sodium atom absorption in the solar light spectrum. The Sodium absorption contains two very closely spaced lines at wavelengths λ =589.0 and 589.6nm, which can be distinguished from their slightly different diffraction angle. Consider the sunlight goes straight down to a standard grating (incident angle $i = 0$) with periodicity *d*=(1/1200) mm, calculate the angles of the 1st order diffraction θ_1 and show that for small wavelength difference $\Delta\lambda$, we have

$$\frac{\Delta\theta}{\theta} = -\frac{\Delta\lambda}{d}\left(1 - \frac{\lambda^2}{d^2}\right)^{-1/2}$$

c) Diffraction gratings are also used in quantum optics experiments to fine tune the laser wavelength at the precision of 10 fm (10^{-14}m) by optical feedback. Let's consider a laser beam is oriented toward the grating and feedback occurs when the first order diffraction goes exactly back toward the laser source $\theta_1 = -\theta_i$. How would you set the incident angle θ_i of a laser beam toward a grating with 1800 lines per meter to stabilize a laser at the wavelength λ =852 nm. Show that you can fine tune the laser wavelength $d\lambda$ by rotating the diffraction grating angle by $d\phi$. Determine the tuning sensitivity $d\lambda/d\phi$ of the laser.