

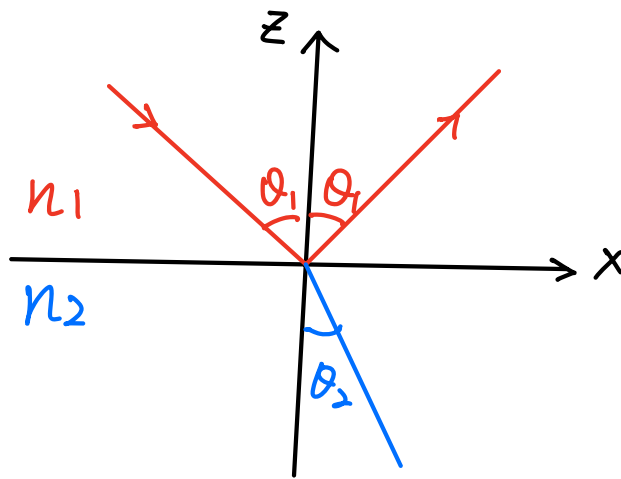
Geometrical Optics

Interface

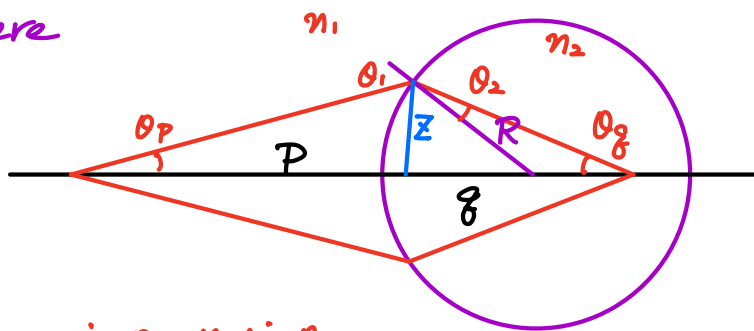
Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

see h.w.



Sphere



$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\tan \theta_p = \frac{z}{p}$$

$$\tan \theta_q = \frac{z}{q}$$

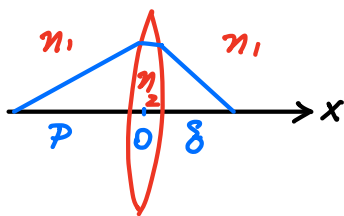
$$\sin(\theta_2 + \theta_q) = \frac{z}{R}$$

$$\theta_1 = \theta_2 + \theta_q + \theta_p$$

$$\begin{aligned} \Rightarrow \theta_2 &= z/R - z/q \\ \Rightarrow \theta_1 &= z/R + z/p \\ \Rightarrow n_1(1/R + 1/p) &= n_2(1/R - 1/q) \\ \Rightarrow \frac{n_1}{p} + \frac{n_2}{q} &= \frac{n_2 - n_1}{R} \end{aligned}$$

$$\sin \theta \approx \tan \theta \approx \theta$$

lens: combination of 2 interfaces from n_1 to n_2 to n_1



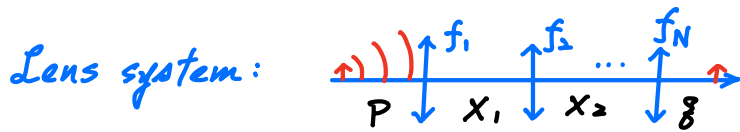
$$\text{1st interface: } \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R_1}$$

$$\text{2nd interface: } \frac{n_2}{-q} + \frac{n_1}{q} = \frac{n_1 - n_2}{-R_2}$$

source is on the right side

$$\text{Add 2 Eqs} \Rightarrow \boxed{\frac{1}{p} + \frac{1}{q} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \equiv \frac{1}{f}} \leftarrow \text{focal length}$$

thin lens formula



① 1st lens: $\frac{1}{p} + \frac{1}{g_1} = \frac{1}{f_1}$

② 2nd lens: $\frac{1}{x_1 - g_1} + \frac{1}{g_2} = \frac{1}{f_2}$

③ Nth lens: $\frac{1}{x_{N-1} - g_{N-1}} + \frac{1}{g} = \frac{1}{f_N}$

p is positive if the source is on the incident side

g is positive if the light converges on the outgoing side.

Applications:

1. Collimation & focusing



$\frac{1}{p} + 0 = \frac{1}{f}$

$0 + \frac{1}{g} = \frac{1}{f}$

2. Magnifier / microscope



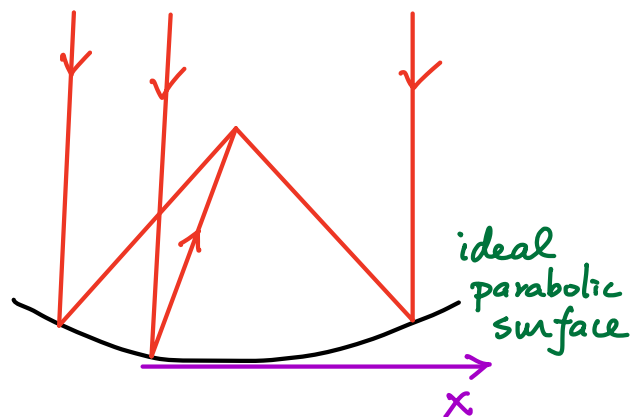
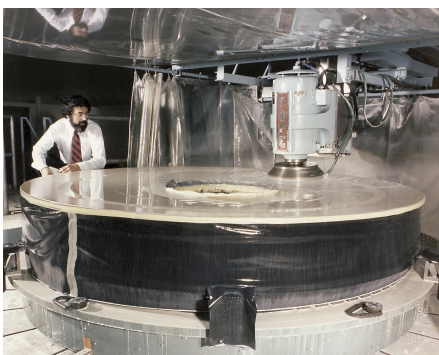
$\frac{1}{p} + \frac{1}{g} = \frac{1}{f}$ Magnification = $\frac{g}{p}$



3. telescope: angular magnification



4. reflecting telescope

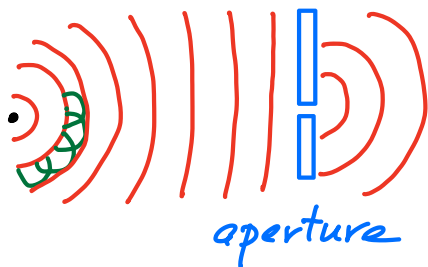


Hubble's mirror flaw 1.3mm too flat on the 2.4m mirror

Diffraction

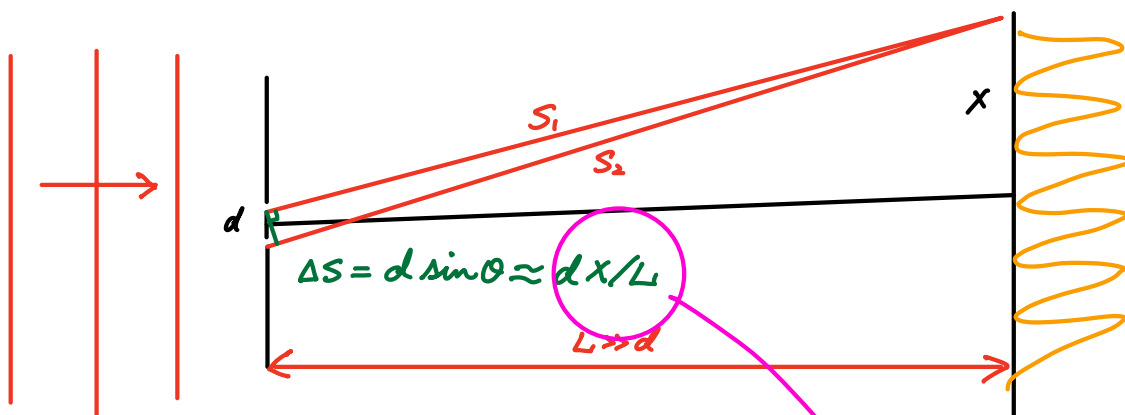
Huygens-Fresnel principle

Every point on a wavefront is itself a source of spherical wave



In spherical coordinate, point source generate $\psi(r, \theta, \phi) = \psi_0 \frac{e^{i(kr - \omega t)}}{r}$ in the far field

Young's double-slit exp



Intensity on the screen $I = \langle |E|^2 \rangle$

$$E = E_1 + E_2 = \frac{1}{S_1} e^{i(kS_1 - \omega t)} + \frac{1}{S_2} e^{i(kS_2 - \omega t)}$$
$$\approx \frac{1}{\langle S \rangle} e^{-i\omega t} \left[e^{ikS_1} + e^{ikS_2} \right]$$
$$S_1 = \sqrt{L^2 + (x - d/2)^2}$$
$$S_2 = \sqrt{L^2 + (x + d/2)^2}$$

Constructive interference: kS_1 and kS_2 are in phase $|e^{i\phi} + e^{i\phi}|^2 = 4$

destructive interference: kS_1 and kS_2 are out of phase $|e^{i\phi} - e^{i\phi}|^2 = 0$

$$I = \langle |E|^2 \rangle = \frac{1}{L^2} |1 + e^{ik(S_2 - S_1)}|^2 = 4I_0 \cos^2 \frac{1}{2} \Delta\phi$$

I_0 : intensity of a single beam

$$\Delta\phi = \phi_2 - \phi_1 = k(S_2 - S_1)$$

max when $\Delta\phi = 0, \pm 2\pi, \pm 4\pi \dots \Rightarrow \Delta S = 0, \pm \lambda, \pm 2\lambda \dots$

min when $\Delta\phi = \pm \pi, \pm 3\pi \dots \Rightarrow \Delta S = \pm \lambda/2, \pm 3\lambda/2 \dots$

Consider max: $S_1 - S_2 = N\lambda$, $N = 0, \pm 1, \pm 2, \dots$

First of all. $|S_1 - S_2| < d$, at most $1 + d/\lambda$ maxima.

If $d < \lambda$ only 1 max at the center.

Consider $\lambda \ll d \ll L$

$$\begin{aligned} |S_1 - S_2| &= \sqrt{L^2 + (x + d/2)^2} - \sqrt{L^2 + (x - d/2)^2} = N\lambda \\ &= L \left[1 + \frac{1}{2} \left(\frac{x + d/2}{L} \right)^2 \right] - L \left[1 + \frac{1}{2} \left(\frac{x - d/2}{L} \right)^2 \right] \\ &= L \frac{1}{2} \frac{1}{L^2} 4x d/2 = \frac{dx}{L} \end{aligned}$$

$\Rightarrow$ location of the n -th max is $x_n = \left(\frac{L}{d} \right) n \lambda$ ← wavelength amplified by $L/d = 10^3 \sim 10^4$!!

$$\text{Intensity} = 4I_0 \cos^2 \frac{k}{2} (S_2 - S_1) = 4I_0 \cos^2 \frac{1}{2} \frac{d}{L} kx$$

Generalized to N -slit.

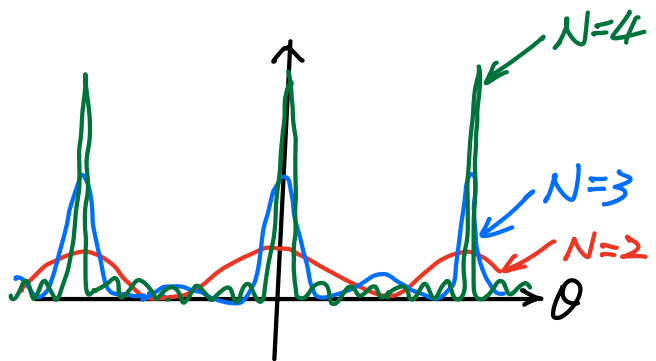
$$E = E_0 + E_1 + \dots + E_{N-1} = E \sum_{j=0}^{N-1} e^{ikS_j} = E \sum e^{ikj d \sin \theta} = E \sum_{j=0}^{N-1} e^{i\alpha j}$$

$\alpha = ikd \sin \theta$

$$= E \frac{\alpha^N - 1}{\alpha - 1} \quad 1 + 2 + 4 + 8 = 16 - 1$$

$$\Rightarrow I = I_0 \frac{\sin^2 N\pi d \sin \theta / \lambda}{\sin^2 \pi d \sin \theta / \lambda}$$

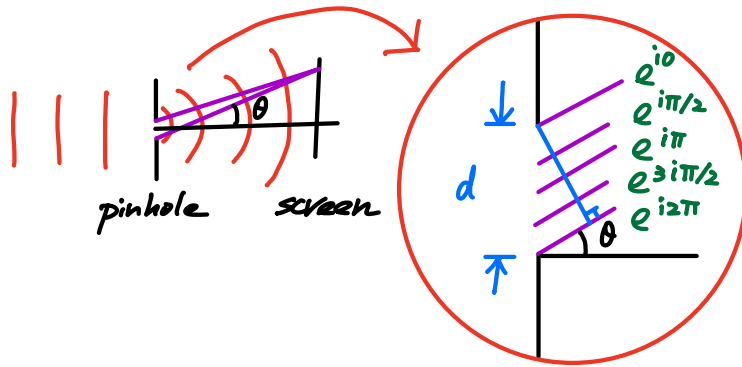
Peak value: $I = N^2 I_0$ N slits give N^2 higher intensity at max.



width $N\pi d \sin \theta / \lambda = \pi \Rightarrow \Delta \theta = \frac{\lambda}{Nd} \pi$ decreases with N .

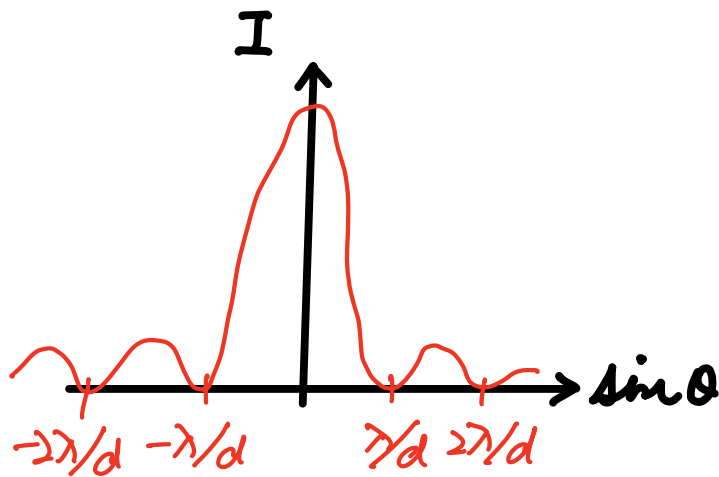
Thus total energy $\int I(\theta) d\theta = N^2 I_0 \frac{\lambda \pi}{Nd} \propto N$ still proportional to # of slits.

Single slit diffraction



$$\text{max: } \theta = 0, d \sin \theta = \pm \lambda/2, \pm 3\lambda/2, \dots$$

$$\text{min: } d \sin \theta = \pm \lambda, \pm 2\lambda, \dots$$



When $d \sin \theta = N\lambda$, a beam with phase $e^{i\theta}$ is always paired with one with $e^{i\theta + i\pi} = -e^{i\theta}$
 $\Rightarrow$ Complete destructive interference.

$$E = \int_{-d/2}^{d/2} A e^{iKS(x)} dx = A e^{iKS_0} \int_{-d/2}^{d/2} e^{-i\frac{\pi}{\lambda} x \sin \theta} dx = A e^{iKS_0} \frac{2}{k \sin \theta} \sin \frac{k d \sin \theta}{2}$$

$$|E|^2 = 4A \frac{\sin^2 \frac{k d \sin \theta}{2}}{k^2 \sin^2 \theta}$$

numerator goes to 0 when $d \sin \theta = 0, \pm \lambda, \pm 2\lambda, \dots$
 denominator goes to 0 when $\sin \theta = 0$.

In the limit of $\theta \rightarrow 0$ $|E|^2 = 4A \lim_{\theta \rightarrow 0} \frac{k^2 d^2 \theta^2 / 4}{k^2 \theta^2} = A d^2$ is actually the max