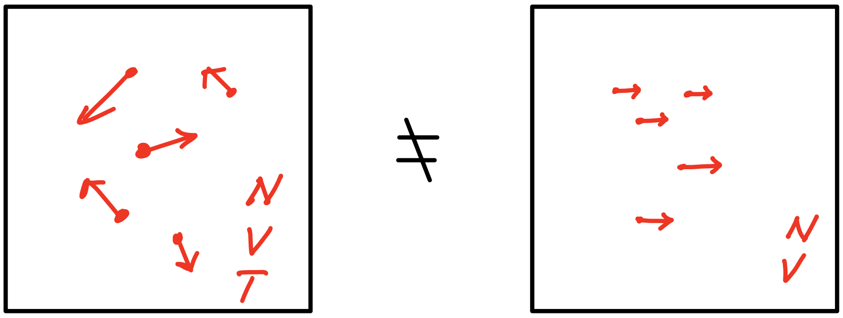
Fundamental Laws of thermodynamics

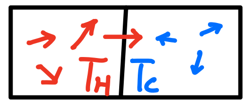
What is heat and how is it different from other forms of energy?



$\langle E_K \rangle = \frac{3}{2} k_B T$ $\quad\quad$ $\langle E_K \rangle = \frac{3}{2} k_B T$

thermal energy is random $\langle \vec{v} \rangle = 0$

2nd, heat only flows from hot side to cold side



until equlibrium is reached $T_H = T_c$

0th law of thermodynamics: A & B in equlibrium, B & C in equlibrium $\Rightarrow$ A & C also in equlibrium. $\quad T_A = T_B = T_c$

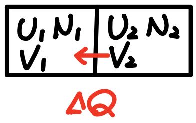
1st law of thermodynamics: energy and heat conservation

$$\Delta U = \underbrace{\Delta Q}_{\text{heat}} + \underbrace{\Delta W}_{\text{work } -pdV} + \underbrace{\Delta C}_{\text{chemical potential } +\mu\Delta N} + \cdots$$ diff forms of energy exchange.

$$dU = TdS - pdV + \mu dN$$ $\quad$ S: entropy.

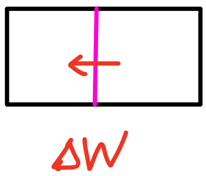
$$\Rightarrow p = -\partial U/\partial V \quad \mu = \partial U/\partial N \quad T \equiv \partial U/\partial S$$

In equilibrium

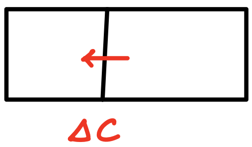


$\Delta U = \Delta(U_1+U_2) = T_1 \Delta S_1 + T_2 \Delta S_2 = 0$

Since $T_1 = T_2$. $\Rightarrow \Delta(S_1+S_2) = 0 \Rightarrow S_1+S_2 = \text{const.}$



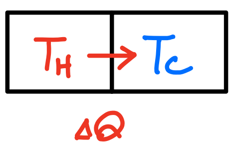
$\Delta U = -P_1 \Delta V_1 - P_2 \Delta V_2 = (P_2 - P_1)\Delta V_1 = 0 \Rightarrow P_1 = P_2 = \text{const.}$



$\Delta U = \mu_1 \Delta N_1 + \mu_2 \Delta N_2 = (\mu_1 - \mu_2)\Delta N_1 = 0 \Rightarrow \mu_1 = \mu_2$

The only difference of heat is that we cannot control entropy flow.

Heat always flows from hot to cold, other processes are flexible.



$\Delta U = T_H \Delta S_H + T_c \Delta S_c = 0$

$\Rightarrow \Delta S = \Delta S_H + \Delta S_c > 0$

2nd law of thermodynamics: A thermodynamical process tends to increase the total entropy of an isolated system.

Fermi: $S(B) - S(A) \geq \int_A^B \frac{dQ}{T}$

Clausius: A transformation whose only final result is to transfer heat from a body at T to another one at a higher T is impossible.

2nd law violates time reversal symmetry, the only law that does not permits time travel. Why?

Third law: $\lim_{T \to 0} S(T) = 0$. $\Rightarrow S$ is going to be positive for all $T > 0$.

Entropy : a measure of randomness of a system.

$$S = k_B \ln W. \quad \text{(Boltzmann)}$$

W: # of states the system can occupy.

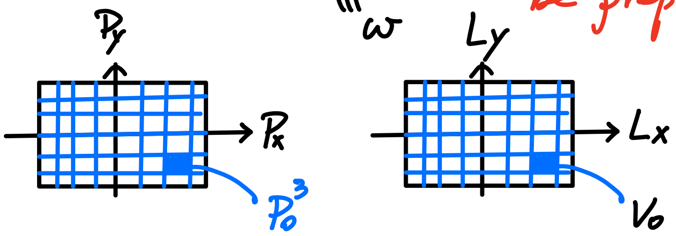
Consider an ideal gas $PV = NkT$. $U = \frac{3}{2} NkT$

$$dU = TdS - PdV \Rightarrow dS = \frac{dU}{T} + \frac{P}{T} dV = \frac{3}{2} Nk \frac{dT}{T} + Nk \frac{dV}{V}$$

$$\Rightarrow S - \cancel{S_0}^{\,0} = \frac{3}{2} Nk \ln \frac{T}{T_0} + Nk \ln \frac{V}{V_0} = Nk \ln \frac{T^{3/2} V}{T_0^{3/2} V_0} = Nk \ln \frac{P^3 V}{P_0^3 V_0}$$

$$\Rightarrow S = k \ln \Bigg(\underbrace{\boxed{\frac{P_x P_y P_z L_x L_y L_z}{P_0^3 \; V_0}}}_{\equiv\, \omega} \Bigg)^N$$

This is the # of ways a particle can be prepared in position & momentum.



$$\Rightarrow S = k \ln \Omega = Nk \ln \omega$$

entropy = $k_B \times \ln$ (# of ways a system can be prepared.)

Quantum mechanics $P_x \Delta x \sim P_y \Delta y \sim P_z \Delta z = h$

$$\Rightarrow \omega = P^3 V / h^3.$$

At $T=0$, system is in the ground state. $\omega = 1$. $\Omega = 1 \Rightarrow S = 0$

$T > 0$ system can be in many states. $\omega > 1$. $\Omega = \omega^N > 1 \Rightarrow S > 0$

Back to definition of temperature $T = \partial U / \partial S = \partial U / \partial k \ln (\text{randomness})^N$

# of available states for a particle $= M$

$$\Rightarrow k_B T = \partial (U/N) / \partial \ln M$$

How much energy will increase when a new degree of freedom is added.

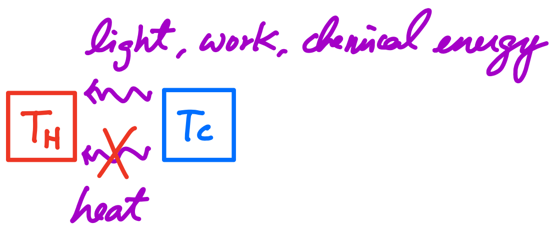
Heat transfer

$$q = -\kappa \nabla T$$ Fourier's law

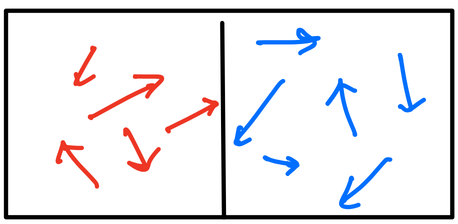
q: heat flux density Energy/area/time

κ: thermal conductivity.

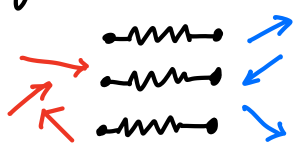
Heat transfer is different from other energy flows

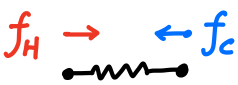


Why can't heat go back to T_H?



We can model the wall as 2 layers of atoms





Why should energy flow in only 1 direction?

The random motion on average only allows heat flows from Hot to Cold!

Other forms of heat transports: convection & radiation.