1. Bose condensation
   Bose statistics
   Phase space density
2. Mean-field approach
   Gross-Pitaevskii equation
3. Experiments
   Time of flight
   Ground state population
   Expanding gas
4. Other interesting experiments
Physics 471 - Introduction to Modern Atomic Physics

Phase space density

Quantum Mechanics:
# of particle in the ground state

Classical mechanics:
# of particle in a unit phase space

\[
\phi = NP(E_0) \\
= n\lambda_{dB}^3 \\
\sim Ne^{-H(x,p)/kT} \left( \frac{dx dp}{\hbar} \right)^3
\]

Magneto-optical trap
Molasses

100 \(\mu\)K \(\rightarrow\) 10 \(\mu\)K

\[
\phi = n\lambda^3 = 10^{-7}
\]
Non-interacting BEC

Atomic wavefunction

\[ \Psi(x_1, x_2, \ldots) = \prod_i \psi_0(x_i) \]

Product state: all atoms occupy ground state.

\[ E_0 \psi_0(r) = -\frac{\hbar^2}{2m} \nabla^2 \psi_0(r) + V(r) \psi_0(r) \]
Assume all atoms have the same wave function

\[ \mu \psi_1 (r) = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(r) + g |\psi_0 (r)|^2 \right) \psi_1 (r) \]

\[ \mu \psi_2 (r) = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(r) + g |\psi_1 (r)|^2 \right) \psi_2 (r) \]

\[ \ldots \]

A “self-consistent” Gross-Pitaevskii equation

\[ \mu \psi (r) = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(r) + g |\psi (r)|^2 \right) \psi (r) \]
Gross-Pitaevskii equation

$$\mu \psi(r) = -\frac{\hbar^2}{2m} \nabla^2 \psi(r) + V(r) \psi(r) + g |\psi(r)|^2 \psi(r)$$

BEC in free space  \( \psi(x) = \text{const.} \)

Chemical potential = zero-point energy + gn

BEC in a harmonic trap

Repulsive atomic interaction \( g>0 \)
\( \Rightarrow \) increasing mean-field energy
\( \Rightarrow \) increasing wavefunction spread
\( \Rightarrow \) increasing potential energy
\( \Rightarrow \) decreasing kinetic energy

Thomas-Fermi approximation (\( E_k \ll V \sim gn \))
Thomas-Fermi approximation: KE << int.

\[ \mu \psi(r) = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(r) + g |\psi(r)|^2 \right) \psi(r) \]

\[ \mu \psi(r) = (V(r) + g n) \psi(r) \]

\[ n(r) = \frac{\mu - V(r)}{g} \quad \text{For } \mu > V(r) \]

\[ N = \int n(r) \, dv \]
Stability of BEC of size $L$ in a box

Kinetic energy = $L^{-2}$
Interaction energy = $a N L^{-3}$

in a harmonic trap

Kinetic energy = $L^{-2}$
Interaction energy = $a N L^{-3}$
Potential energy = $L^2$
Finite temperature

Total particle number $N = N_0$ (condensed) + $N'$ (uncondensed)

$$N' = \sum_{i=\mu+1}^{\infty} n_i$$

$$= \int d\varepsilon \, \rho(\varepsilon) \frac{1}{e^{\varepsilon/kT} - 1}$$

$$= N(T/T_c)^{x+1}$$

Density of state: $\rho(\varepsilon) \sim \varepsilon^x$
Experiment: Expanding thermal gas and expanding condensate

3 \times 10^6 \text{ atoms in an anisotropic magnetic trap}

\begin{align*}
\frac{1}{2} m v_i^2 &= \frac{1}{2} kT \\
\frac{1}{2} m v_i^2 &= \frac{1}{4} \hbar \omega_i
\end{align*}

\begin{align*}
&\text{isotropic expansion} & \text{anisotropic expansion}
\end{align*}

\begin{align*}
&\text{Boltzmann gas} & \text{condensate}
\end{align*}

\begin{align*}
&\text{Time of flight} & \text{Camera} \\
&\text{Laser} \rightarrow \text{atoms} & \text{CCD}
\end{align*}

100 \mu m \times 5 \mu m
0.5 \text{ to } 1 \mu K
TOF: free expansion of cold gas in vacuum

Light sheet

Absorption imaging
BEC as a coherent source of matterwave (1995~2001)

- Matter waves interference (MIT group, 1997)
- Vortices in Bose-Einstein Condensation (JILA group, 2000)
- Matter wave laser (MPQ group, 2000)
- Quantum phase transition in optical lattice (Mainz/MPQ group, 2002)
Trapped BEC's

BEC's after an expansion time \( t \)

\[
\lambda = \frac{h}{m \Delta v} = \frac{ht}{md}
\]

M. R. Andrews et. al.
Science 275, ff. 637, 1997