# P452 Quiz 1

Your name:______________

April 2026

## Reference: Quantum Gates & Bloch Vector

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix},$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \vec{b} = (\langle X \rangle, \langle Y \rangle, \langle Z \rangle).$$

**Instructions:** Circle the answer for the following 10 questions in 15 minutes.

1. Which gate rotates the Bloch vector from the $+x$ direction $\vec{b} = (1, 0, 0)$ to the $+y$ direction $(0, 1, 0)$?
   **A)** $X$ gate **B)** $Y$ gate **C)** $S$ gate **D)** $H$ gate

2. Consider the **W-state** $|W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$. If the first qubit is measured to be 0, what is the normalized state of the remaining two qubits?
   **A)** Product state $|00\rangle$ **B)** Bell state $\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$
   **C)** GHZ state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ **D)** Mixed state of $|01\rangle$ and $|10\rangle$

3. If a hardware coupling map shows no direct connection between $q_0$ and $q_2$, what is the minimum number of **SWAP** gates needed to move the state of $q_2$ to a position adjacent to $q_0$?
   **A)** 1 **B)** 2 **C)** 3 **D)** 4

4. Starting from the state $|000\rangle$, you apply a Hadamard ($H$) gate to the first qubit, then a CNOT gate using the first qubit as control and the second as target ($q_0 \to q_1$), and finally a CNOT gate from the second to the third ($q_1 \to q_2$). What is the resulting final state?
   **A)** $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ **B)** $\frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$
   **C)** $\frac{1}{\sqrt{2}}(|010\rangle + |110\rangle)$ **D)** $\frac{1}{\sqrt{2}}(|000\rangle + |011\rangle)$

5. When measuring a Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, if the first qubit is measured to be 0, what is the probability that the second qubit will also be measured as 0?
   **A)** 0% **B)** 25% **C)** 50% **D)** 100%

6. To simulate a 2-site Fermi-Hubbard model with 1 spin-up ($\uparrow$) and 1 spin-down ($\downarrow$) fermion using the Jordan-Wigner transformation, how many qubits are required, and what are the dimensions of the physical Hilbert space vs. the qubit Hilbert space?
   **A)** 2 qubits; Physical dim = 4, Qubit dim = 4 **B)** 4 qubits; Physical dim = 4, Qubit dim = 16
   **C)** 4 qubits; Physical dim = 16, Qubit dim = 16 **D)** 2 qubits; Physical dim = 2, Qubit dim = 4

7. Consider the Fermi-Hubbard model $H = -J\sum(c^\dagger c + h.c.) + U\sum n_\uparrow n_\downarrow$. If $U \to \infty$, what happens to the probability of double occupancy?
   **A)** Increases linearly with $J$ **B)** Constant at 50%
   **C)** Suppressed to near zero **D)** Oscillates at Rabi frequency

8. Why is "Trotterization" necessary for simulating the time evolution $e^{-i(H_J+H_U)\tau}$?
   **A)** Hamiltonian is non-Hermitian **B)** $H_J$ and $H_U$ terms do not commute
   **C)** To convert time to binary format **D)** Quantum computation only supports single-and two-qubit gates

9. In the 2-qubit XY model ($H = J(X_1X_2 + Z_1Z_2)$), which of the following is true for the state $|01\rangle$?
   **A)** It is an eigenstate **B)** It is a superposition and will evolve
   **C)** It is the ground state **D)** Energy eigenvalue is $+2J$

10. What is the minimum number of 1-qubit and 2-qubit gates required to prepare an 10-qubit system such that a measurement on any single qubit changes the probability distribution of the remaining 9 qubits?
    **A)** 1 Hadamard, 9 CNOTs **B)** 10 Hadamards, 10 CNOTs

## Correct Answers & Explanations

1. **C) $S$ gate**
The $S$ gate (Phase gate) performs a 90° ($\pi/2$) rotation around the $z$-axis. Starting with the Bloch vector $(1, 0, 0)$, which represents the state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, applying the $S$ gate results in $S|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$. This state corresponds to the $+y$ direction $(0, 1, 0)$. The $H$ gate would instead rotate $(1, 0, 0)$ to the $+z$ direction $(0, 0, 1)$.

2. **B) Bell state $\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$**
The W-state is $\frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$. Measuring $q_0 = 0$ collapses the state by removing the $|100\rangle$ term. This leaves $|010\rangle + |001\rangle$. Once normalized, and ignoring the measured qubit, the state of the remaining two qubits is the entangled Bell state $\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$.

3. **A) 1**
In a linear chain $q_0 - q_1 - q_2$, there is no direct connection between $q_0$ and $q_2$. Performing one SWAP gate between $q_1$ and $q_2$ moves the state originally at $q_2$ to the $q_1$ position, which is now adjacent to $q_0$.

4. **A) $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$**
Applying $H$ to $q_0$ creates $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$. The first CNOT ($q_0 \to q_1$) creates the Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$. The final CNOT ($q_1 \to q_2$) entangles the third qubit, resulting in the GHZ (or Cat) state.

5. **D) 100%**
The $|\Phi^+\rangle$ state is perfectly correlated. If the first qubit is 0, the wavefunction collapses entirely to $|00\rangle$, meaning the second qubit must also be 0.

6. **B) 4 qubits; Physical dim = 4, Qubit dim = 16**
For a 2-site Hubbard model, each site requires two qubits (one for spin-up, one for spin-down), totaling 4 qubits. While the restricted physical subspace ($1\uparrow$, $1\downarrow$) has only 4 possible configurations, the total Hilbert space available to 4 qubits is $2^4 = 16$.

7. **C) Suppressed to near zero**
In the Fermi-Hubbard model, $U$ represents the energy penalty for two fermions occupying the same site. As $U \to \infty$, the cost of double occupancy becomes prohibitive, effectively suppressing it (Mott insulator physics).

8. **B) $H_J$ and $H_U$ terms do not commute**
Trotterization is used to approximate the operator $e^{A+B} \approx (e^{A/n}e^{B/n})^n$. This is necessary because for the Hubbard model, the kinetic energy (hopping) and potential energy (interaction) operators do not commute, so $e^{H_J+H_U} \neq e^{H_J}e^{H_U}$.

9. **B) It is a superposition and will evolve**
   The state $|01\rangle$ is not an eigenstate of the XY Hamiltonian $H = J(X_1X_2 + Z_1Z_2)$. It can be expressed as a linear combination of the Bell states $|\Psi^+\rangle$ and $|\Psi^-\rangle$, which have different eigenvalues, leading to time evolution.

10. **A) 1 Hadamard, $N-1$ CNOTs**
    To create a GHZ state where every qubit is entangled with the rest, a single Hadamard creates the initial superposition and a chain of $N-1$ CNOT gates propagates that entanglement to every other qubit in the system.