Physics 452 - Quantum Optics and Quantum Gasses Cheng Chin

HOMEWORK 2 (Due: 10/16/2017)

1. Single channel scattering in square well

Given a square well potential in the spherical coordinate $V(r < r_0) = -\frac{\hbar^2 q^2}{2\mu} \equiv -D$ and $V(r > r_0) = 0$, the incoming spherical wave is e^{-ikr}/r and the outgoing wave is Se^{ikr}/r, where $S = e^{2i\delta}$ is the scattering matrix and δ is the s-wave scattering phase shift.

A. show that the scattering phase shift is $\delta = -kr_0 + \tan^{-1} \frac{k \tan\sqrt{q^2 + k^2}\, r_0}{\sqrt{q^2 + k^2}}$.

B. Determine scattering length a in the low scattering energy limit $k \to 0$.

C. Plot the scattering phase shift and scattering length vs. the depth D.

D. What is the value of scattering phase shift when the scattering length diverges?

2. Bound states and potential resonance

Continue 1 with the square potential. Here we will look into the connection between the scattering length and the bound state.

A. When we increase D from zero, show that the condition for the potential to support one more bound state is the same as the condition you derived in 1 D.

B. In particular, when the scattering length is very large and positive $a >> r_0$, show that there is a weakly bound state near the dissociation threshold with energy

$$E = -\frac{\hbar^2}{2\mu(a - r_0)^2}.$$

C. According to molecular spectroscopy, there are 153 bound states of a diatomic cesium molecule Cs_2 in the singlet potential. What is the singlet scattering phase shift of two colliding cesium atoms in the low temperature limit?